Open games in practice

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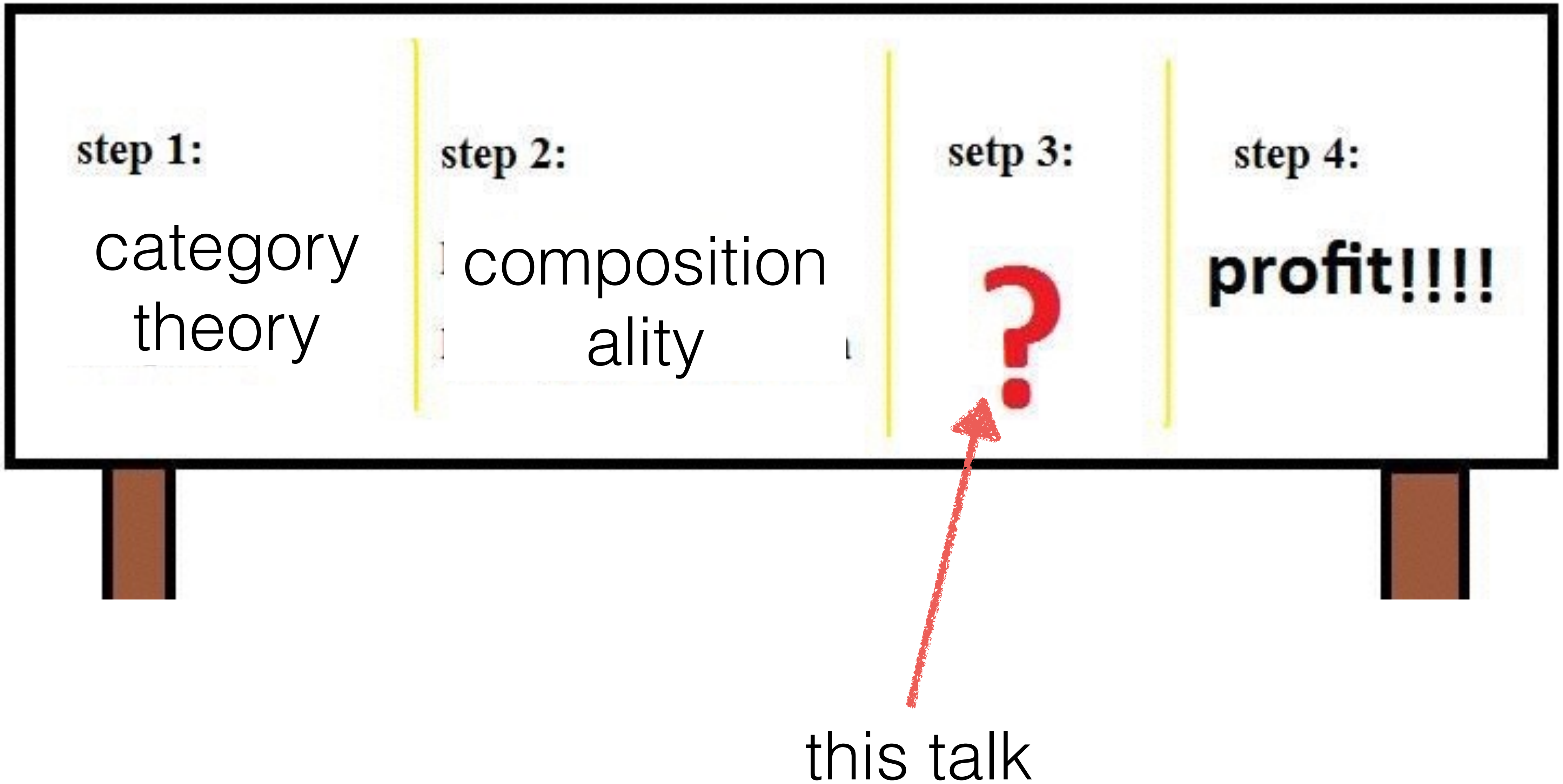

step 1:
category theory
step 2:
composition ality
setp 3:
?
step 4:
profit!!!!
this talk

Zanzi #BlackLivesMatter, Silence is Complicity @tangled_z... · 2m

You forgot the prelude slide of "If you're not familiar with catamorphic zygozorphisms already then I'm sorry I don't think I can convey the intuition in just a few slides, and I suggest getting a head start on the refreshments in the break room"

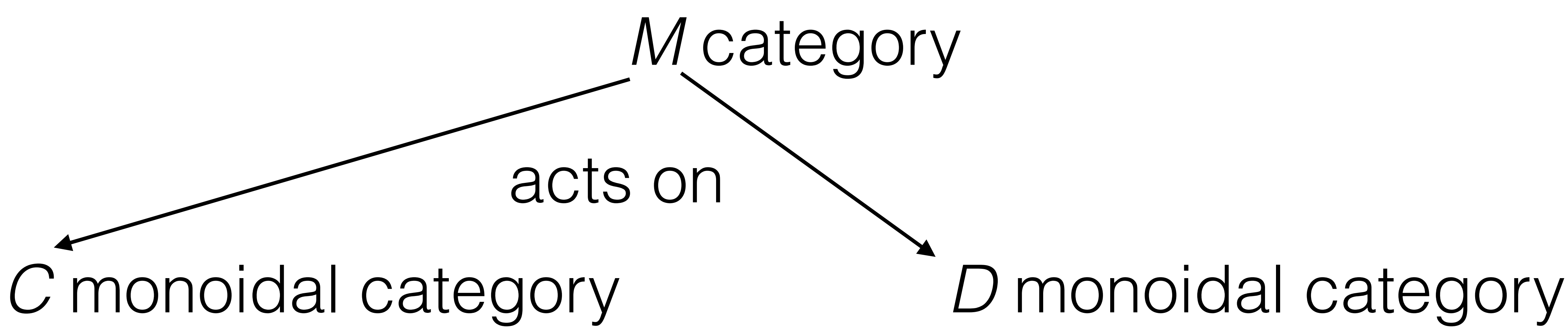
1 1

julesh
@_julesh_

Replying to @tangled_zans

This but unironically, anybody who missed the keynote on mixed optics will probably be in for a rough time

Mixed optics recap



$$\mathbf{Optic}\left(\begin{pmatrix} S \\ T \end{pmatrix}, \begin{pmatrix} A \\ B \end{pmatrix}\right) = \int^{M \in \mathcal{M}} \mathcal{C}(S, M \cdot A) \times \mathcal{D}(M \cdot B, T)$$

Optics in this talk

D = (f.s.) probability monad on **Set**

forwards category = kleisli category of *D*
= category of (f.s.) Markov kernels

acts by “lift”

backwards category = kleisli category of

$$T(X) = \mathbb{R}^N \to \mathcal{D}(\mathbb{R}^N \times X)$$

state monad transformer, state = payoff vector
applied to *D*

(~~nasty~~ rather nice hack)

Open games recap

(play goes forwards)

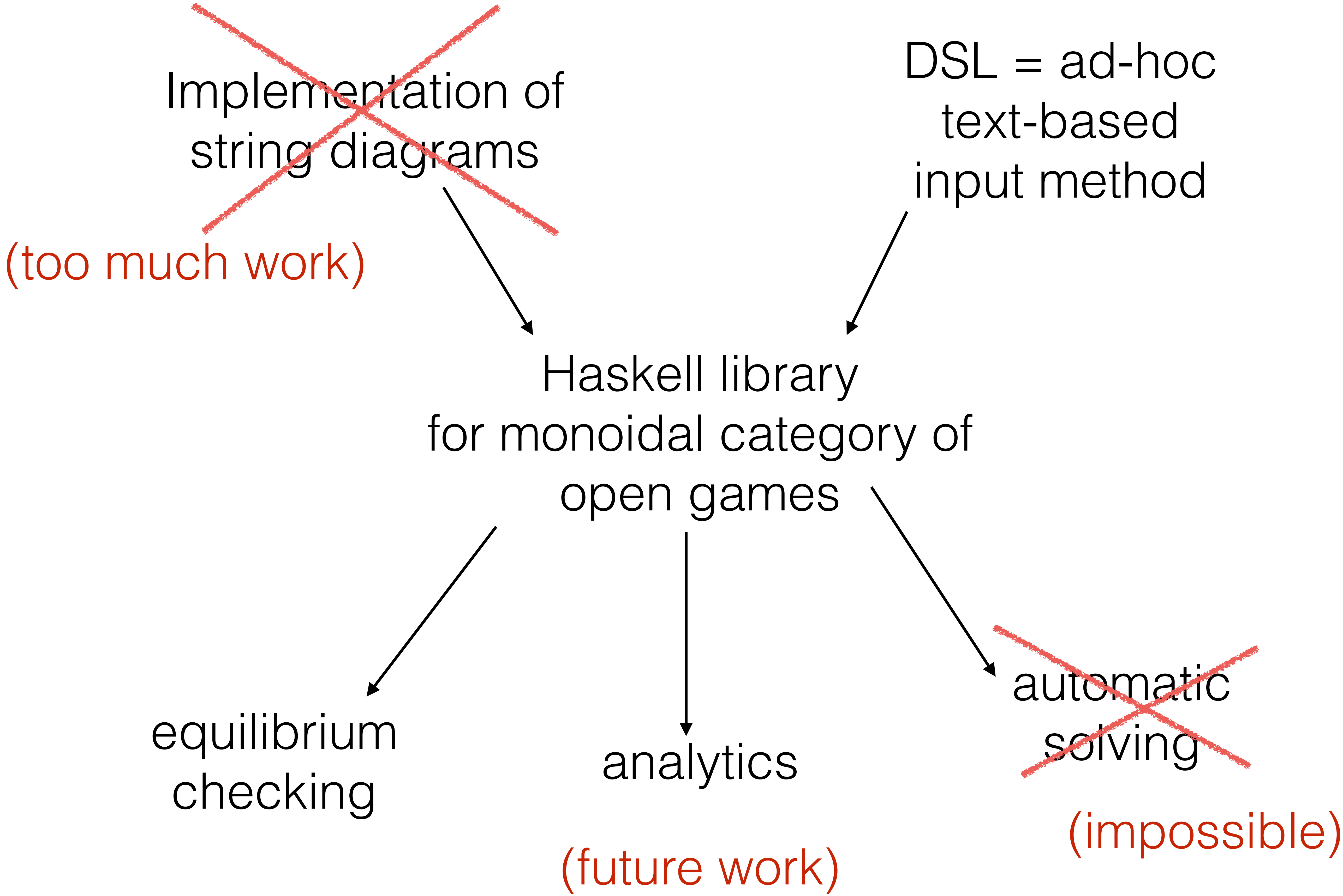
An open game $\begin{pmatrix} S \\ T \end{pmatrix} \to \begin{pmatrix} A \\ B \end{pmatrix}$ consists of:

(payoffs go backwards)

1. A set ∑ of strategy profiles
2. A ∑-indexed family of optics $\begin{pmatrix} S \\ T \end{pmatrix} \to \begin{pmatrix} A \\ B \end{pmatrix}$
3. For each ∑ and context, a "valuation"

(list of deviations)

$$\overline{\mathbf{Optic}}\left(\begin{pmatrix} S \\ T \end{pmatrix}, \begin{pmatrix} A \\ B \end{pmatrix}\right) = \int^{\Theta} \mathcal{D}(\Theta \times S) \times (A \to T(B))$$


Implementation of string diagrams
(too much work)
DSL = ad-hoc text-based input method
Haskell library for monoidal category of open games
equilibrium checking
analytics
(future work)
automatic solving
(impossible)

The pipeline

- Code in a domain specific language (DSL) + auxiliary Haskell code describes game

  ↓ DSL-to-Haskell compiler

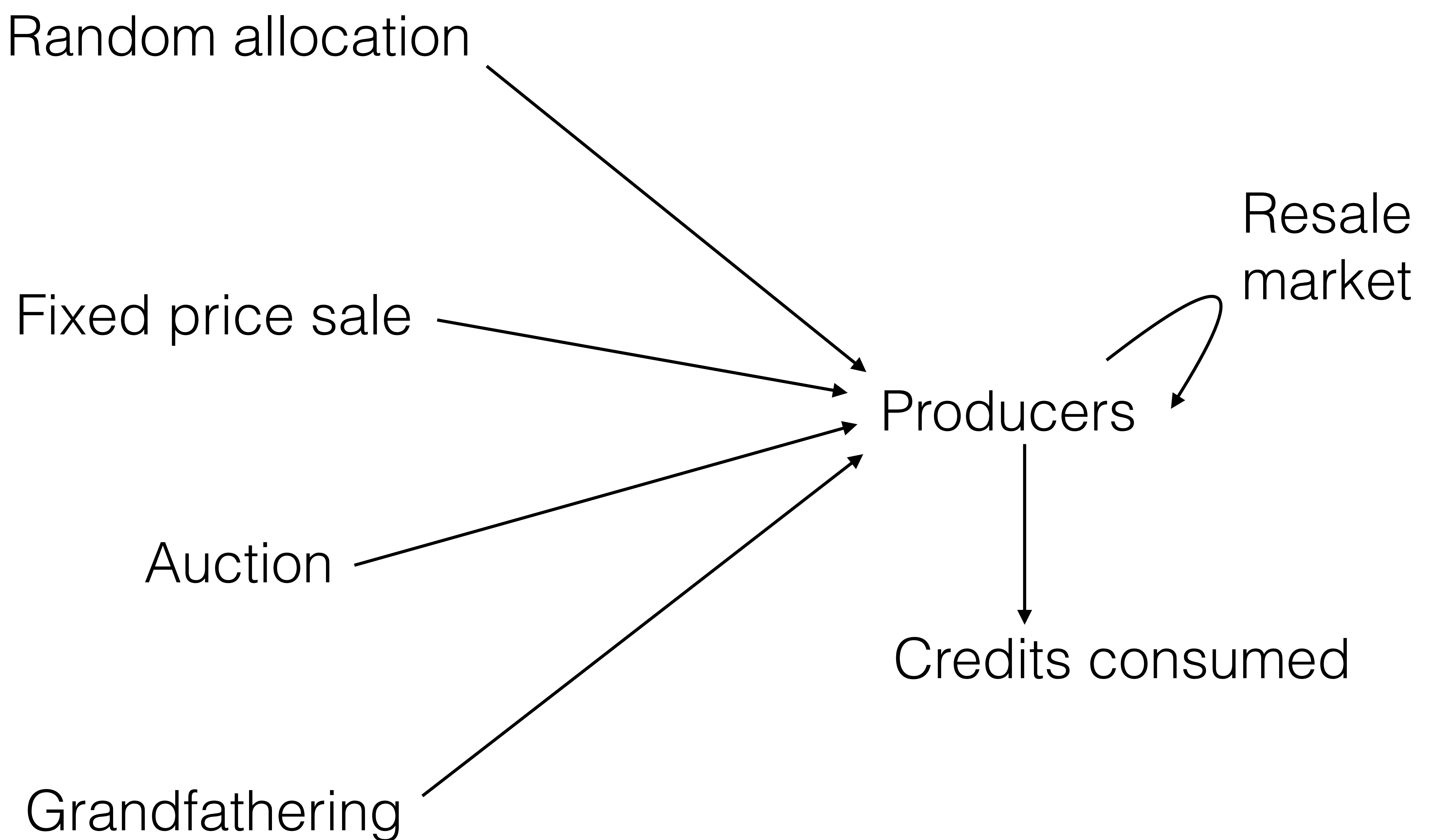
- Haskell code importing open games backend library

  ↓ Load in interactive Haskell prompt (GHCi)

- Interactive model

Worked example

- 1 carbon credit = legal right to emit 1000 kg CO_2
- Total credits capped by emissions target
- How to allocate credits to producers?
- Mechanism design: We would like to design the rules to produce a “good” outcome (e.g. avoiding perverse incentives)
- Our goal: rapid prototyping of models


Random allocation
Fixed price sale
Auction
Grandfathering
Producers
Resale
market
Credits consumed
= flow of credits

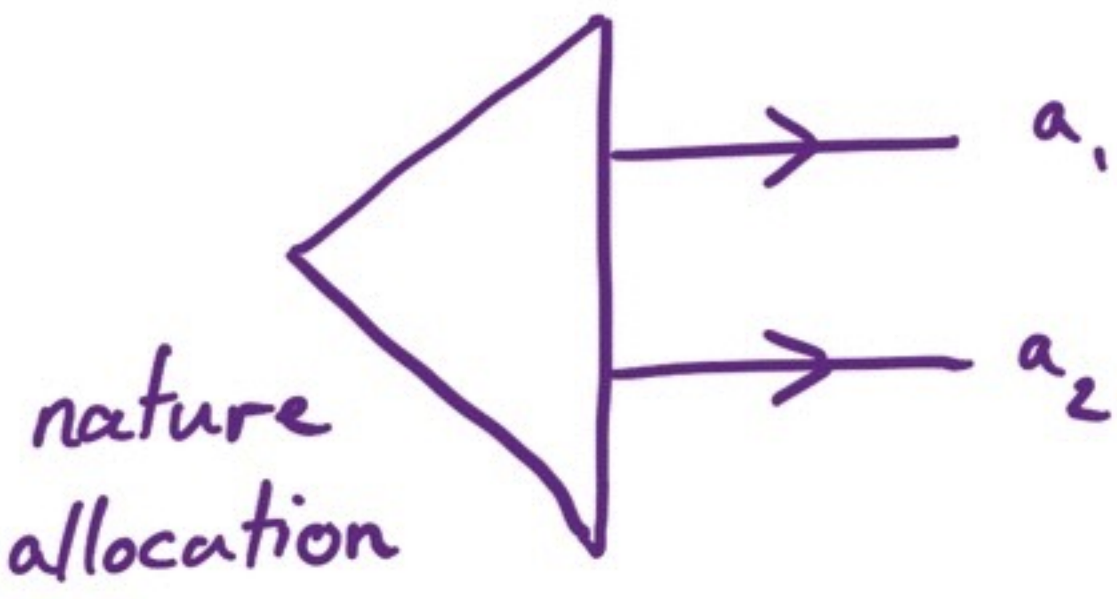
Baby steps

- Fix 2 firms,
- Fix 2 credits to be allocated

Random allocation

```
randomAllocation = \game ->
  a1, a2 <- nature allocation
  returnG -< a1, a2
```

lift a distribution



Supplementary Haskell:

```
allocation :: Stochastic (Int, Int)
allocation = do a1 <- uniform [0..2]
                return (a1, 2 - a1)
```

Fixed price allocation

Inputs: Private valuations

Information available to player

```
fixedAllocation = \game v1, v2 ->
  ask1 <- decision "player1" [0..2] -< v1 | -3*a1
  ask2 <- decision "player2" [0..2] -< v2 | -3*a2
  a1, a2 <- function allocatePermits -< ask1, ask2
  returnG -< a1, a2
```

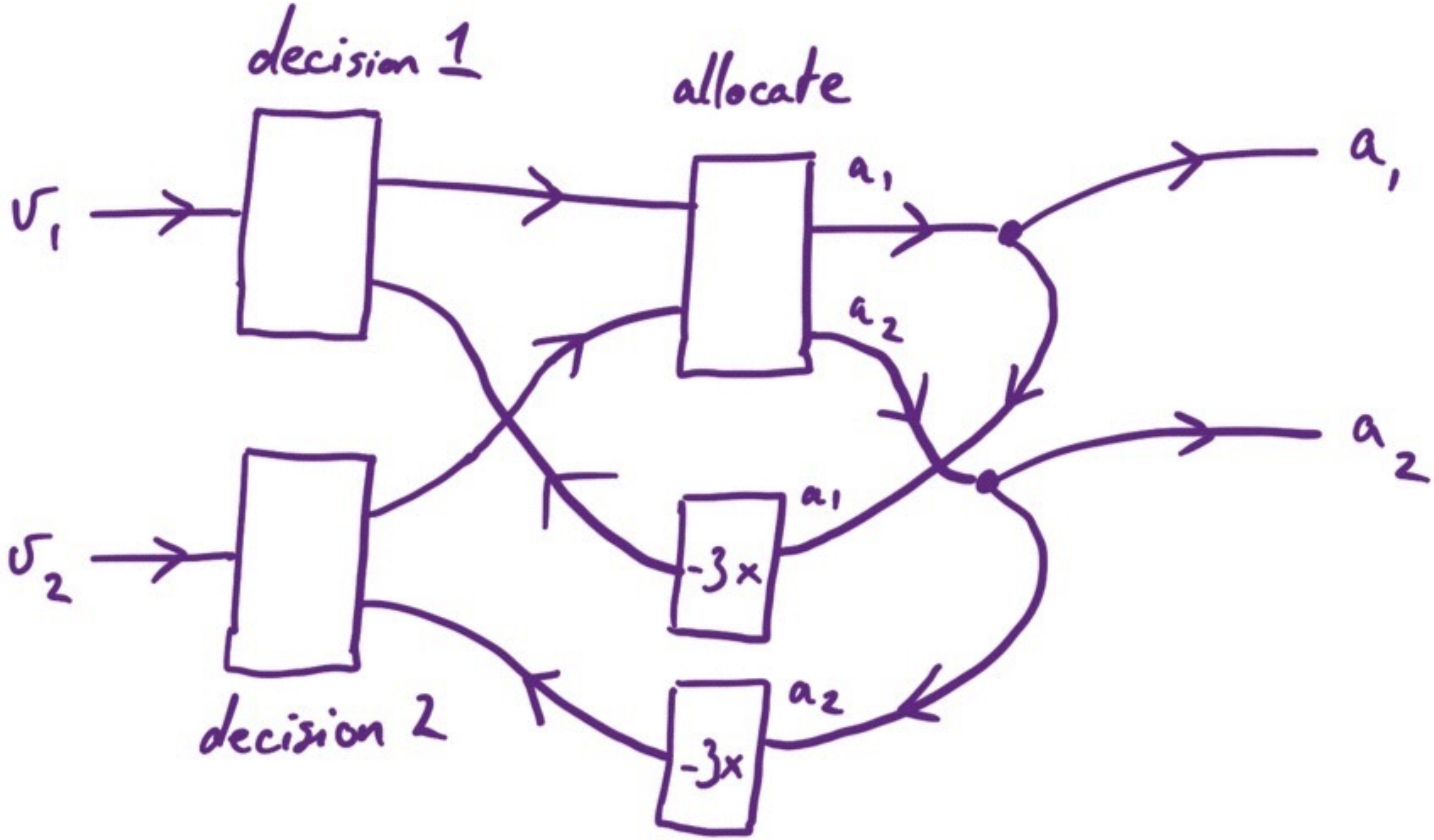
Decision variable

Output: allocations

Payoff from decision

Supplementary Haskell:

```
allocatePermits :: (Int, Int) -> (Int, Int)
allocatePermits (ask1, ask2)
  | (ask1 + ask2 <= 2) = (ask1, ask2)
  | (otherwise)        = (1, 1)
```



decision 1
allocate
v_1
v_2
a_1
a_2
a_1
a_2
a_1
-3x
a_2
-3x
decision 2

VCG auction

```
vcgAllocation = \game v1, v2 ->
  bid1 <- decision “player1” [0..5] -< v1 | -pays1
  bid2 <- decision “player2” [0..5] -< v2 | -pays2
  a1, pays1, a2, pays2 <- function auctioneer -< bid1, bid2
  returnG -< a1, a2
```

Supplementary Haskell (2nd price auction):

```
auctioneer :: (Int, Int) -> (Int, Double, Int, Double)
auctioneer (bid1, bid2)
  | (bid1 == bid2) = (1, bid2/2, 1, bid1/2)
  | (bid1 >  bid2) = (2, bid2,   0, 0)
  | (bid1 <  bid2) = (0, 0,      2, bid1)
```

Production game

Input: valuations + allocated credits

```
production = \game v1, v2, a1, a2 ->
  c1 <- decision “player1” [0..a1] -< v1, a1 | v1*c1
  c2 <- decision “player2” [0..a2] -< v2, a2 | v2*c2
  returnG -< a1 - c1, a2 - c2
```

consumption decision

Output: remaining credits

faked dependent type

The story so far:

```
game1 = \game ->
  v1 <- nature (uniform [1..5])
  v2 <- nature (uniform [1..5])
  a1, a2 <- vcgAllocation -< v1, v2
  _, _ <- production -< v1, v2, a1, a2
```

(At this stage we can do equilibrium analysis)

Resale market

(skipped for time)

Hooking it together

```
game2 = \game ->
  v1 <- nature (uniform [1..5])
  v2 <- nature (uniform [1..5])
  a1, a2 <- vcgAllocation -< v1, v2
  b1, b2 <- production -< v1, v2, a1, a2
  c1, c2 <- resaleMarket -< v1, v2, b1, b2
  _, _ <- production -< v1, v2, c1, c2
```

Notes

- Equilibrium analysis skipped for time!
- … but it was the purpose of the whole exercise
- Repository: `https://github.com/jules-hedges/open-games-hs`
- This example: `https://github.com/jules-hedges/open-games-hs/blob/permitSale/src/OpenGames/Examples/PermitSale/PermitSale.hs`